中國人民大學
RENMIN UNIVERSITY OF CHINA

复杂批次系统智能学习控制

4. RIM: Fading Channels

沈栋
分布式人工智能实验室
中国人民大学数学学院
中国人民大学应用数学研究中心
金融计算与数字工程教育部工程研究中心

OUTLINE

研究背景
Background

统计已知
Known Statistics

统计未知
Unknown Statistics

统计变化
Varying Statistics

Plant model

$$x_k(t+1) = Ax_k(t) + Bu_k(t)$$
$$y_k(t) = Cx_k(t)$$

Reference model

$$x_d(t+1) = Ax_d(t) + Bu_d(t)$$
$$y_d(t) = Cx_d(t)$$

ILC scheme

$$u_{k+1}(t) = u_k(t) + a_k L e_k(t+1)$$

Assumptions

- Relative degree
- Initial state
- System noise

Fading channel model

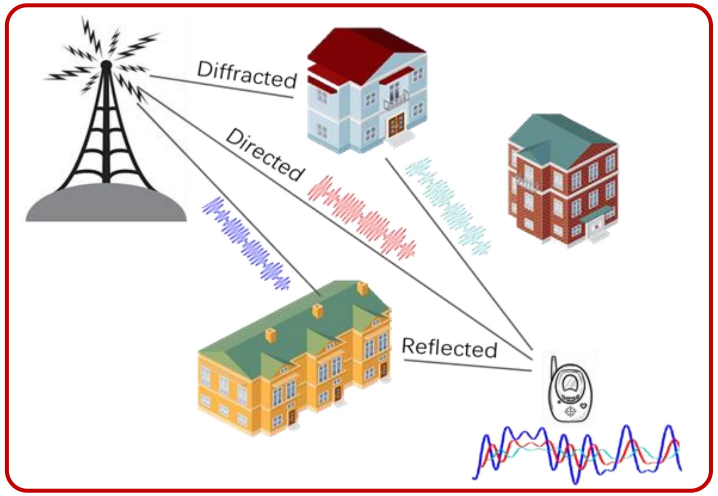
$$m_k^{\circ}(t) = \mu_k(t) m_k(t) + \nu_k(t)$$

Multiplicative randomness

$$\mathbb{P}(\mu_k(t) > 0) = 1$$
$$\mu = \mathbb{E}[\mu_k(t)], \qquad \sigma^2 = \mathbb{E}[(\mu_k(t) - \mu)^2]$$

Additive randomness

$$\mathbb{E}[\nu_k(t)] = 0, \qquad \mathbb{E}\left[\nu_k(t)\nu_k^T(t)\right] = R_t$$



Extensions

- High-dimension
- Correlation

Output fading

$$y_k^{\circ}(t) = \mu_k(t) y_k(t) + v_k(t)$$

Correction

$$y_k'(t) = \mu^{-1} y_k^{\circ}(t)$$

$$\begin{aligned} y_k'(t) &= \mu^{-1}\mu_k(t) y_k(t) + \mu^{-1} v_k(t) \\ &= y_k(t) - \left(1 - \frac{\mu_k(t)}{\mu}\right) y_k(t) + \mu^{-1} v_k(t) \end{aligned}$$

ILC scheme

$$u_{k+1}(t) = u_k(t) + a_k L \epsilon_k(t+1)$$

$$\epsilon_k(t) = y_d(t) - y_k'(t)$$

$$a_k > 0, \sum_{k=1}^{\infty} a_k = \infty, \sum_{k=1}^{\infty} a_k^2 < \infty$$

Convergence Theorem: Consider system (1) with **output** fading channels and apply the ILC scheme with a decreasing gain sequence. If the direction regulation matrix L is designed such that all eigenvalues of LCB have positive real parts, then the input sequence generated by the ILC scheme converges asymptotically in the mean-square sense to the desired value $u_d(t)$. That is, $\mathbb{E}[\|\boldsymbol{u}_d - \boldsymbol{u}_k\|^2] \to 0$.

Input fading

$$u_k^{\circ}(t) = \mu_k(t)u_k(t) + \nu_k(t)$$

Correction

$$u_k'(t) = \mu^{-1}u_k^{\circ}(t)$$

$$u_k'(t) = \mu^{-1}\mu_k(t)u_k(t) + \mu^{-1}\nu_k(t)$$
$$= u_k(t) - \left(1 - \frac{\mu_k(t)}{\mu}\right)u_k(t) + \mu^{-1}\nu_k(t)$$

ILC scheme

$$u_{k+1}(t) = u_k(t) + a_k L e_k(t+1)$$
$$e_k(t) = y_d(t) - y_k(t)$$
$$a_k > 0, \sum_{k=1}^{\infty} a_k = \infty, \sum_{k=1}^{\infty} a_k^2 < \infty$$

Convergence Theorem: Consider system (1) with **input** fading channels and apply the ILC scheme with a decreasing gain sequence. If the direction regulation matrix L is designed such that all eigenvalues of LCB have positive real parts, then the input sequence generated by the ILC scheme converges asymptotically in the mean-square sense to the desired value $u_d(t)$. That is, $\mathbb{E}[\|\boldsymbol{u}_d - \boldsymbol{u}_k\|^2] \to 0$.

System dynamics

$$x_k(t+1) = Ax_k(t) + Bu_k'(t)$$
$$= Ax_k(t) + \mu^{-1}B\mu_k(t)u_k(t) + \mu^{-1}Bv_k(t)$$

Problem

This introduces unpredictable randomness into the system dynamics.

Averaging technique

$$\mathbb{A}[m_k] = \frac{1}{n}\sum_{i=1}^{n} m_{k+1-i}$$

$$u_k^*(t) = \mathbb{A}[u_k'(t)] = \frac{1}{n}\sum_{i=1}^{n} u_{k+1-i}'(t)$$

ILC scheme

$$u_{k+1}(t) = \mathbb{A}[u_k(t)] + a_k L e_k(t+1)$$

$$x_k(t+1) = Ax_k(t) + B\mathbb{A}[u_k'(t)]$$
$$= Ax_k(t) + \mu^{-1}B\frac{1}{n}\sum_{i=1}^{n}\mu_{k+1-i}(t)u_{k+1-i}(t)$$
$$+\mu^{-1}B\frac{1}{n}\sum_{i=1}^{n}v_{k+1-i}(t)$$

Elimination of Fading Effect

$$\mathbb{E}\left[\left(\frac{1}{n}\sum_{i=1}^{n}v_{k+1-i}(t)\right)\left(\frac{1}{n}\sum_{i=1}^{n}v_{k+1-i}(t)\right)^T\right]$$
$$= \frac{1}{n^2}\mathbb{E}\left[\sum_{i=1}^{n}v_{k+1-i}(t)v_{k+1-i}^T(t)\right]$$
$$= \frac{1}{n^2}nR_t = \frac{1}{n}R_t \xrightarrow[n\to\infty]{} 0$$

Convergence Theorem: Consider system (1) with **input fading** channels and apply the ILC scheme with a decreasing gain sequence satisfying $a_j = a_k\big(1 + O(a_k)\big), \forall j = k-1, \dots, k-n+1$ and the regular stochastic approximation condition. If the direction regulation matrix L is designed such that all eigenvalues of LCB have positive real parts, then the input sequence generated by the ILC scheme converges asymptotically in the mean-square sense to the desired value $u_d(t)$. That is, $\mathbb{E}[||\boldsymbol{u}_d - \boldsymbol{u}_k||^2] \to 0$.

Problem

How to deal with the effect of the faded input signal on system dynamics?

Additional conflict

Learning ability vs **Tracking ability**

Learning ability → convergence rate of the proposed learning algorithm

Tracking ability → final tracking precision to the desired reference

Average operators

Moving-average operator

$$\mathbb{A}^M[\boldsymbol{u}_k] = \frac{1}{m}\sum_{i=1}^{m} \boldsymbol{u}_{k+1-i}$$

General-average operator

$$\mathbb{A}^G[\boldsymbol{u}_k] = \frac{1}{k}\sum_{i=1}^{k} \boldsymbol{u}_i$$

Forgetting-average operator

$$\mathbb{A}^F[\boldsymbol{u}_k] = \frac{1-\gamma}{1-\gamma^k}\sum_{i=1}^{k} \gamma^{i-1}\boldsymbol{u}_{k+1-i}$$

Plant model

$$x_k(t+1) = Ax_k(t) + Bu_k(t)$$
$$y_k(t) = Cx_k(t)$$

→ $$\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{u}_k + \boldsymbol{d}_k$$

$$\boldsymbol{H} = \begin{bmatrix} p_1 & 0 & \cdots & 0 \\ p_2 & p_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p_n & p_{n-1} & \cdots & p_1 \end{bmatrix}$$

Reference model

$$x_d(t+1) = Ax_d(t) + Bu_d(t)$$
$$y_d(t) = Cx_d(t)$$

→ $$\boldsymbol{y}_d = \boldsymbol{H}\boldsymbol{u}_d + \boldsymbol{d}_d$$

Iterative learning control

$$u_{k+1}(t) = u_k(t) + \alpha L e_k(t+1)$$

→ $$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + \alpha \boldsymbol{L}\boldsymbol{e}_k$$

Convergence condition

$$\rho(I - \alpha \boldsymbol{L}\boldsymbol{H}) < 1$$

Corrected input

$$\boldsymbol{u}_k^c = \mu^{-1}\Theta_k \boldsymbol{u}_k$$

$$= \boldsymbol{u}_k - \Phi_k(\boldsymbol{u}_d - \boldsymbol{u}_k) + \Phi_k \boldsymbol{u}_d$$

Error dynamics

$$\boldsymbol{e}_k = \boldsymbol{H}(\boldsymbol{u}_d - \boldsymbol{u}_k^c)$$

$$= \boldsymbol{H}\tilde{\boldsymbol{u}}_k + \boldsymbol{H}\Phi_k\tilde{\boldsymbol{u}}_k - \boldsymbol{H}\Phi_k\boldsymbol{u}_d$$

Control revision

Moving-average operator

$$\boldsymbol{y}_k^M = \boldsymbol{H}\mathbb{A}^M[\boldsymbol{u}_k^c] + \boldsymbol{d}_k$$

$$\boldsymbol{e}_k^M = \boldsymbol{y}_d - \boldsymbol{y}_k^M = \boldsymbol{H}\left(\boldsymbol{u}_d - \mathbb{A}^M[\boldsymbol{u}_k^c]\right)$$

$$\boldsymbol{u}_{k+1} = \mathbb{A}^M[\boldsymbol{u}_k] + \alpha h \boldsymbol{e}_k^M$$

ILC with MA

General-average operator

$$\boldsymbol{u}_{k+1} = \mathbb{A}^G[\boldsymbol{u}_k] + \alpha h \boldsymbol{e}_k^G$$

ILC with GA

Forgetting-average operator

$$\boldsymbol{u}_{k+1} = \mathbb{A}^F[\boldsymbol{u}_k] + \alpha h \boldsymbol{e}_k^F$$

ILC with FA

Moving-average case

Convergence Theorem: Consider system (1) with **input fading** channels and apply the ILC scheme with **MA**. If the learning gain α is sufficiently small and h satisfies $p_1 h > 0$, then the input error $\boldsymbol{u}_d - \boldsymbol{u}_k$ converges to a neighborhood of zero in the mean-square sense

$$\lim_{k\to\infty} \mathbb{E}[\|\boldsymbol{u}_d - \boldsymbol{u}_k\|^2] \le \kappa(\boldsymbol{O}) \frac{\eta_2 \alpha^2 \sigma_\mu^2 \|\boldsymbol{u}_d\|^2}{(1-\rho) m \mu^2}$$

where $\kappa(\boldsymbol{O})$ is the condition number of a positive-definite matrix $\boldsymbol{O}$, $\boldsymbol{O}$ is the solution of the Lyapunov equation $\boldsymbol{H}^T\boldsymbol{O} + \boldsymbol{O}\boldsymbol{H} = h^{-1}I$, η_2 is a suitable constant, and ρ is the contract factor determined by step size α.

Learning ability

$$\mathbb{E}[\tilde{\boldsymbol{u}}_k^T \boldsymbol{O} \tilde{\boldsymbol{u}}_k] \le \rho^{\left\lfloor \frac{k}{m} \right\rfloor} \max_{1\le i\le m} \{\mathbb{E}[\tilde{\boldsymbol{u}}_i^T \boldsymbol{O} \tilde{\boldsymbol{u}}_i]\} + \frac{1-\rho^{\left\lfloor \frac{k}{m} \right\rfloor}}{1-\rho} \eta_2 \delta$$

$$\rho = 1 - c\alpha + \eta_1 \alpha^2$$

Tracking ability

$$\mathbb{E}\left[\left\|\boldsymbol{e}_k^M\right\|^2\right] \le 2 \max_{1\le i\le m} \left\{\mathbb{E}\left\|\mu^{-1}\boldsymbol{H}\Theta_{k+1-i}\tilde{\boldsymbol{u}}_{k+1-i}\right\|^2\right\} + \frac{2}{m}\mathbb{E}\|\boldsymbol{H}\Phi_{k+1-i}\boldsymbol{u}_d\|^2$$

Moving-average case

Convergence Theorem: Consider system (1) with **input fading** channels and apply the ILC scheme with **GA**. If the learning gain α is sufficiently small and h satisfies $p_1 h > 0$, then the average input error $\mathbb{A}^G[\boldsymbol{u}_d - \boldsymbol{u}_k]$ converges to zero in the mean-square sense

$$\lim_{k\to\infty} \mathbb{E}\left[\left\|\mathbb{A}^G[\tilde{\boldsymbol{u}}_k]\right\|^2\right] = 0.$$

A quick note

$$\mathbb{E}\left[\left\|\mathbb{A}^G[\tilde{\boldsymbol{u}}_k]\right\|^2\right] \le \mathbb{A}^G[\mathbb{E}\|\tilde{\boldsymbol{u}}_k\|^2] \longrightarrow 0$$

Learning ability

$$\mathbb{E}\left[\left(\mathbb{A}^G[\tilde{\boldsymbol{u}}_{k+1}]\right)^T O\left(\mathbb{A}^G[\tilde{\boldsymbol{u}}_{k+1}]\right)\right] \le (1-\rho_k)\mathbb{E}\left[\left(\mathbb{A}^G[\tilde{\boldsymbol{u}}_k]\right)^T O\left(\mathbb{A}^G[\tilde{\boldsymbol{u}}_k]\right)\right] + \delta_k \frac{\alpha^2 \lambda_{\max}(\boldsymbol{O})\sigma_\mu^2 \|\boldsymbol{u}_d\|^2}{\mu(k+1)k}$$

$$\rho_k = \frac{c_1\alpha - c_2\alpha^2}{k+1} - \frac{c_3\alpha^2}{(k+1)^2} \backsim \frac{\tau}{k}$$

$\Rightarrow$

$\tau > 1$	$O(k^{-1})$
$\tau = 1$	$O(k^{-\tau}\log k)$
$\tau < 1$	$O(k^{-\tau})$

$$\delta_k = \frac{c_4}{k+1} + c_5$$

Tracking ability

$$\mathbb{E}\left[\left\|\boldsymbol{e}_k^G\right\|^2\right] \le 2\mathbb{E}\left[\left\|\mathbb{A}^G[\mu^{-1}\boldsymbol{H}\Theta_k\tilde{\boldsymbol{u}}_k]\right\|^2\right] + 2\mathbb{E}\left[\left\|\mathbb{A}^G[\boldsymbol{H}\Phi_k\boldsymbol{u}_d]\right\|^2\right]$$

$$\mathbb{E}\left[\left\|\mathbb{A}^G[\boldsymbol{H}\Phi_k\boldsymbol{u}_d]\right\|^2\right] \le \frac{1}{k} \times \omega\mathbb{E}[\Phi_k^2] \longrightarrow 0$$

Forgetting-average case

Convergence Theorem: Consider system (1) with **input fading** channels and apply the ILC scheme with **FA**. If the learning gain α is sufficiently small and h satisfies $p_1 h > 0$, then the average input error $\mathbb{A}^F[\boldsymbol{u}_d - \boldsymbol{u}_k]$ converges to a neighborhood of zero in the mean-square sense

$$\lim_{k\to\infty} \mathbb{E}\left[\left\|\mathbb{A}^F[\tilde{\boldsymbol{u}}_k]\right\|^2\right] \leq \kappa(\boldsymbol{O}) \frac{\varpi \|\boldsymbol{u}_d\|^2 \sigma_\mu^2}{\mu},$$

where

$$\varpi = \frac{[3c_1(1-\gamma)^2 + c_6(1-\gamma)]\alpha^2}{c_3\alpha - [3(c_1+c_2)(1-\gamma) + (c_4+c_5)]\alpha^2}.$$

Learning ability

$$\mathbb{E}[L_{k+1}] \leq (1-\rho_k)\mathbb{E}[L_k] + \delta_k \frac{\|\boldsymbol{u}_d\|^2 \lambda_{\max}(\boldsymbol{O})\sigma_\mu^2}{\mu}$$

$$\rho_k = c_3\varphi_k\alpha - 3(c_1+c_2)\varphi_k^2\alpha^2 - (c_4+c_5)\varphi_k\alpha^2$$

$$\delta_k = (3c_1\varphi_k + c_6)\alpha^2\varphi_k\varphi_{k-1}$$

$$\varphi_k = \frac{1-\gamma}{1-\gamma^{k+1}}$$

geometrical

$$1 > \varphi_1 > \cdots > \varphi_k > \varphi_{k+1} > \cdots > 1-\gamma$$

Forgetting-average case

Convergence Theorem: Consider system (1) with **input fading** channels and apply the ILC scheme with **FA**. If the learning gain α is sufficiently small and h satisfies $p_1 h > 0$, then the average input error $\mathbb{A}^F[\boldsymbol{u}_d - \boldsymbol{u}_k]$ converges to a neighborhood of zero in the mean-square sense

$$\lim_{k\to\infty} \mathbb{E}\left[\left\|\mathbb{A}^F[\tilde{\boldsymbol{u}}_k]\right\|^2\right] \le \kappa(\boldsymbol{O}) \frac{\varpi\|\boldsymbol{u}_d\|^2\sigma_\mu^2}{\mu},$$

where

$$\varpi = \frac{[3c_1(1-\gamma)^2 + c_6(1-\gamma)]\alpha^2}{c_3\alpha - [3(c_1+c_2)(1-\gamma) + (c_4+c_5)]\alpha^2}.$$

Tracking ability

$$\mathbb{E}\left[\left\|\boldsymbol{e}_k^F\right\|^2\right] \le 2\mathbb{E}\left[\left\|\mathbb{A}^F[\mu^{-1}\boldsymbol{H}\Theta_k\tilde{\boldsymbol{u}}_k]\right\|^2\right] + 2\mathbb{E}\left[\left\|\mathbb{A}^F[\boldsymbol{H}\Phi_k\boldsymbol{u}_d]\right\|^2\right]$$

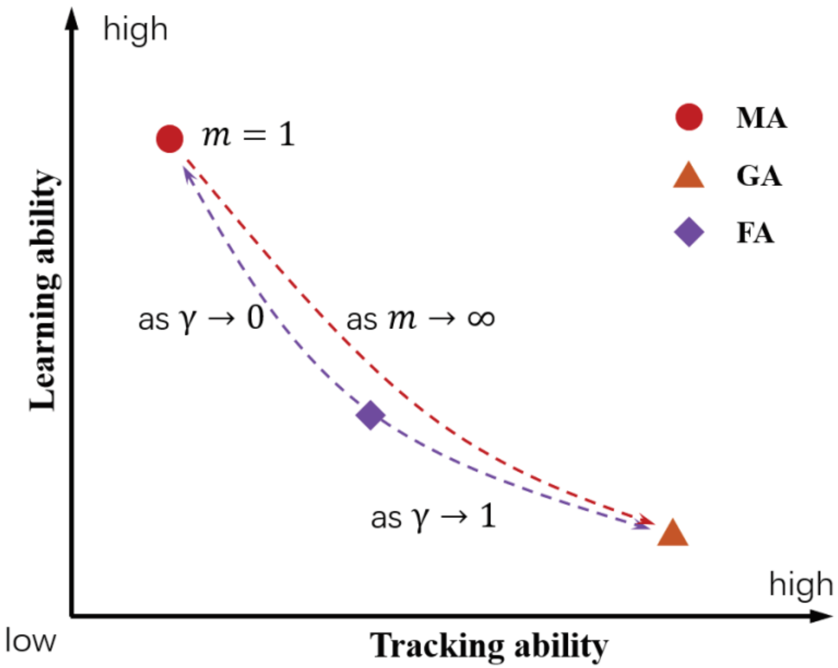
A quick note

$$\mathbb{E}[\mathbb{A}^F[\Phi_k]] = 0$$

$$\mathbb{E}\left[\left(\mathbb{A}^F[\Phi_k]\right)^2\right] = \varphi_{k-1}^2 \sum_{i=1}^{k} \gamma^{2(i-1)}\mathbb{E}[\Phi_{k+1-i}^2] \longrightarrow \frac{1-\gamma}{1+\gamma} \times \frac{\sigma_\mu^2}{\mu} I$$

- Intuitively explain of why the FA-based scheme cannot guarantee zero convergence of the input error

统计已知 ⊙ 收敛结果

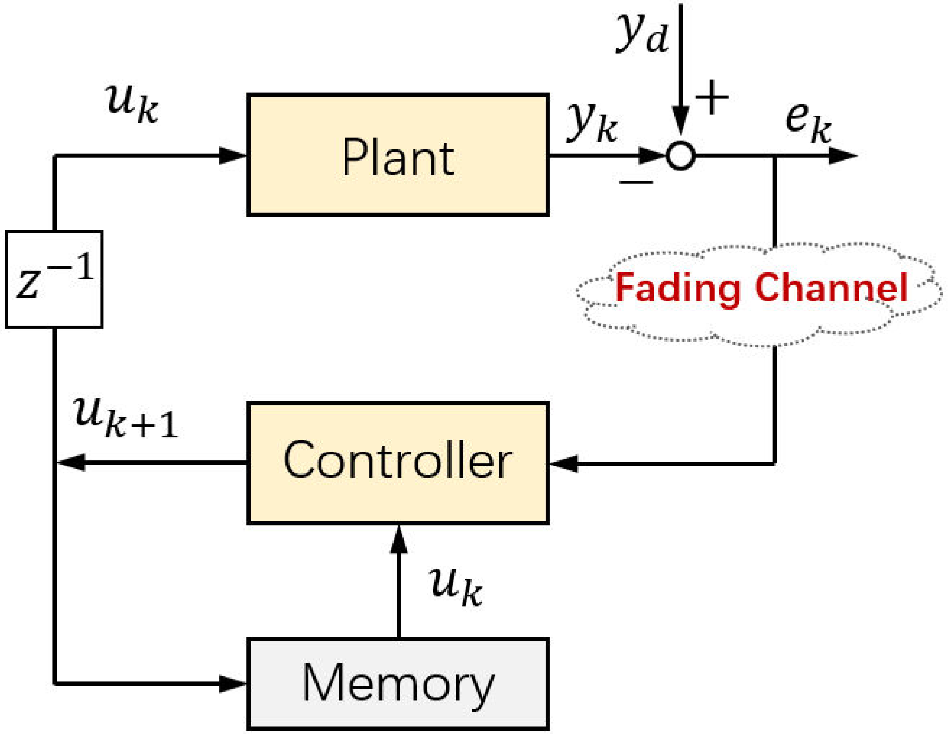


- FA connects MA and GA
- As γ approaches zero, FA becomes MA with $m = 1$
- As γ approaches one, FA becomes GA
- As m approaches infinity, MA becomes GA

Problem

How to deal with the issue if the statistics are unknown?

Configuration transformation



Faded error

$$\boldsymbol{e}_k = \boldsymbol{y}_d - \boldsymbol{y}_k = \boldsymbol{H}\tilde{\boldsymbol{u}}_k$$

$$\boldsymbol{e}_k^{\circ} = (\Theta_k \otimes I_q)\boldsymbol{e}_k + \boldsymbol{v}_k$$

$$\Theta_k = \mathrm{diag}\{\mu_k(1), \mu_k(2), \dots, \mu_k(N)\}$$

Technical settings

$$\boldsymbol{\delta}_k = [\delta_k^1, \delta_k^2, \dots, \delta_k^l]^T \qquad |\delta_k^j| < a$$

$$\bar{\boldsymbol{\delta}}_k = \left[\frac{1}{\delta_k^1}, \frac{1}{\delta_k^2}, \dots, \frac{1}{\delta_k^l}\right]^T \qquad \left|\frac{1}{\delta_k^j}\right| < b \qquad \mathbb{E}\left[\frac{1}{\delta_k^j}\right] = 0$$

Gradient-estimate-based ILC

$$\boldsymbol{u}_{2k+1} = \boldsymbol{u}_{2k} + c_k \boldsymbol{\delta}_k$$

$$\boldsymbol{u}_{2k+2} = \boldsymbol{u}_{2k} - \frac{a_k \bar{\boldsymbol{\delta}}_k}{c_k}\left(\|\boldsymbol{e}_{2k+1}^{\circ}\|^2 - \|\boldsymbol{e}_{2k}^{\circ}\|^2\right)$$

$$a_k > 0, a_k \to 0, \sum_{k=1}^{\infty} a_k = \infty$$

$$c_k > 0, c_k \to 0, \sum_{k=1}^{\infty} \left(\frac{a_k}{c_k}\right)^2 = \infty$$

Convergence Theorem: Consider system (1) and apply the gradient estimate-based ILC scheme. Assume that Assumptions 1-3 hold, and the generated input sequence is bounded, then the input sequence $\boldsymbol{u}_k$ converges to $\boldsymbol{u}_d$ asymptotically along the iteration axis. Accordingly, the output $\boldsymbol{y}_k$ will converge to the desired reference $\boldsymbol{y}_d$ asymptotically.

Gradient-estimate-based ILC

$$\boldsymbol{u}_{2k+1} = \boldsymbol{u}_{2k} + \tau\boldsymbol{\delta}_k$$

$$\boldsymbol{u}_{2k+2} = \boldsymbol{u}_{2k} - \frac{a_k \bar{\boldsymbol{\delta}}_k}{\tau}\left(\|\boldsymbol{e}^{\circ}_{2k+1}\|^2 - \|\boldsymbol{e}^{\circ}_{2k}\|^2\right)$$

$$a_k > 0, a_k \to 0, \sum_{k=1}^{\infty} a_k = \infty$$

Convergence Theorem: Consider system (1) and apply the **variant** gradient estimate-based ILC scheme, where the parameter satisfies $\sum_{k=1}^{\infty} a_k^2 < \infty$. Assume that Assumptions 1-3 hold, and the generated input sequence is bounded. Then, the input error $\tilde{\boldsymbol{u}}_k$ converges to a bounded zone around zero, where the upper bound is linearly dependent on τ. Accordingly, the tracking error is also linearly bounded by the parameter τ.

Fading channel model

$$m_k^\circ(t) = \mu_k(t) m_k(t)$$

Pilot signal

$$\theta_k \equiv 1$$

$$\theta_k^\circ = \mu_k(*)\theta_k = \mu_k(*)$$

Iterative estimation

$$\hat{\mu}_k = \frac{1}{k}\sum_{i=1}^{k}\theta_i^\circ$$

Variants

$$\hat{\mu}_k = \frac{k-1}{k}\hat{\mu}_{k-1} + \frac{1}{k}\theta_k^\circ$$

$$\hat{\mu}_k = (1-\gamma^k)\left(\frac{1}{k}\sum_{i=1}^{k}\theta_i^\circ\right) + \gamma^k \bar{\mu}$$

Estimation property

$$\mathbb{E}[\hat{\mu}_k] = \frac{1}{k}\sum_{i=1}^{k}\mathbb{E}[\theta_i^\circ] = \frac{1}{k}\sum_{i=1}^{k}\mathbb{E}[\mu_i(*)] = \mu$$

$$\mathrm{Var}(\hat{\mu}_k) = \mathbb{E}[\hat{\mu}_k - \mu]^2 = \mathbb{E}\left[\frac{1}{k}\sum_{i=1}^{k}\mu_i(*) - \mu\right]^2$$

$$= \frac{1}{k^2}\mathbb{E}\left[\sum_{i=1}^{k}(\mu_i(*) - \mu)^2\right]$$

$$+ \frac{1}{k^2}\mathbb{E}\left[\sum_{1\le i<j\le k}(\mu_i(*) - \mu)\big(\mu_j(*) - \mu\big)\right]$$

$$= \frac{1}{k}\sigma^2$$

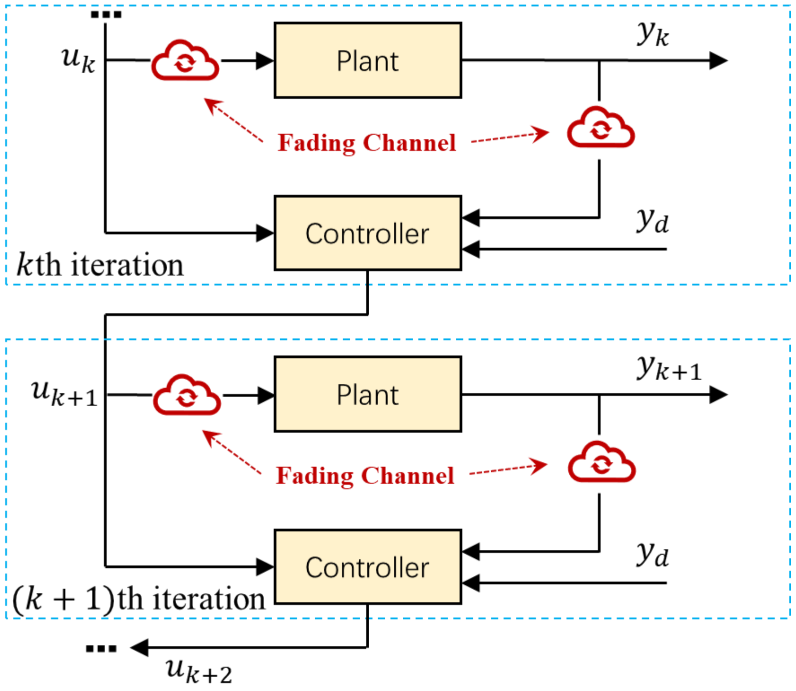
Coupled effect in analysis

$$y_k^\circ(t) = \mu_k(t) y_k(t)$$

$$y_k'(t) = \hat{\mu}_k^{-1} y_k^\circ(t) = \hat{\mu}_k^{-1} \mu_k(t) y_k(t) = y_k(t) + \left(\frac{\mu_k(t)}{\mu} - 1\right) y_k(t) + \left(\frac{1}{\hat{\mu}_k} - \frac{1}{\mu}\right) \mu_k(t) y_k(t)$$

Not zero-mean

Convergence Theorem: Consider system (1) with **output fading** channels and apply the ILC scheme with the iterative estimation of the fading mean. If the direction regulation matrix L is designed such that all eigenvalues of LCB have positive real parts and step size a_k satisfies the stochastic approximation condition, then the input sequence generated by the ILC scheme converges in the mean-square sense to the desired value $u_d(t)$. That is, $\mathbb{E}[\|\boldsymbol{u}_d - \boldsymbol{u}_k\|^2] \to 0$.



$$u_k^{\circ}(t) = \mu_k^{\text{in}}(t)u_k(t) \qquad u_k'(t) = \left(\hat{\mu}_k^{\text{in}}\right)^{-1} u_k^{\circ}(t)$$

$$x_k(t+1) = Ax_k(t) + B\left(\hat{\mu}_k^{\text{in}}\right)^{-1}\mu_k^{\text{in}}(t)u_k(t)$$
$$y_k(t) = Cx_k(t)$$

$$\theta_k^{\circ,\text{out}} = \mu_k^{\text{out}}(*) \qquad y_k^{\circ}(t) = \mu_k^{\text{out}}(t)y_k(t)$$

$$\hat{\mu}_k^{\text{out}} = \frac{1}{k}\sum_{i=1}^{k}\theta_i^{\circ,\text{out}}$$
$$\epsilon_k(t) = y_d(t) - \left(\hat{\mu}_k^{\text{out}}\right)^{-1}\mu_k^{\text{out}}(t)y_k(t)$$

$$u_{k+1}(t) = u_k(t) + a_k L\epsilon_k(t+1)$$

$$\theta_{k+1}^{\circ,\text{in}} = \mu_{k+1}^{\text{in}}(*) \qquad \hat{\mu}_{k+1}^{\text{in}} = \frac{1}{k+1}\sum_{i=1}^{k+1}\theta_i^{\circ,\text{in}}$$

Problem

What to do if the mean and variance varies along the iteration axis?

Fading model

$$m_k^\circ(t) = \mu_k(t) m_k(t) + \nu_k(t)$$

Varying statistics

$$\mu_k = \mathbb{E}[\mu_k(t)] \qquad \tau_k = \mathbb{E}[(\mu_k(t) - \mu_k)^2] \qquad 0 < a \le \mu_k \le b$$

$$\nu_k = \mathbb{E}\left[\nu_k^{(i)}(t)\right] \qquad \delta_k = \mathbb{E}\left[\left(\nu_k^{(i)}(t) - \nu_k\right)^2\right] \qquad \tau_k \le \tau \qquad \delta_k \le \delta$$

Conventional case

$$\nu_k \equiv 0$$

Testing signals

$$\theta_k^\circ(t) = \mu_k'(t)\theta_k(t) + \nu_k'(t) = \mu_k'(t) + \nu_k'(t)$$

Mean estimate

$$\hat{\mu}_k = \frac{1}{n}\sum_{t=1}^{n} \theta_k^\circ(t)$$

Mean inverse estimate **(MIE)**

$$0 \le \varepsilon \le \mu_k^{-1} \qquad M_{1,h} = \frac{1}{h}\sum_{t=1}^{h} \theta_k^\circ(t)$$

$$M_{2,h} = \frac{1}{h}\sum_{t=1}^{h} \left(\theta_k^\circ(t)\right)^2$$

If $M_{1,h} > 0$, define

$$P_h = 1 - \sqrt{\frac{1}{h}\sum_{t=1}^{h}\left(1 - \omega_h \theta_k^\circ(t)\right)}$$

with $\omega_h = \min\left\{\frac{1}{hM_{1,h}}, \frac{M_{1,h}}{M_{2,h}}, \varepsilon\right\}$

Otherwise, take $P_h = 1/h$ and $\omega_h = \varepsilon/h$

MIE

$$\widehat{\gamma_k} = \frac{\omega_h}{q_N}\prod_{t=1}^{N}\left(1 - \omega_h \theta_k^\circ(t)\right)$$

Received output

$$y_k^\circ(t) = \mu_k(t) y_k(t) + \nu_k(t)$$

Corrected output

$$\hat{y}_k(t) = \widehat{\gamma_k} y_k^\circ(t)$$
$$= y_k(t) + \left(\frac{\mu_k(t)}{\mu_k} - 1\right) y_k(t)$$
$$+ (\widehat{\gamma_k} - \mu_k^{-1}) \mu_k(t) y_k(t) + \widehat{\gamma_k} \nu_k(t)$$

ILC scheme

$$u_{k+1}(t) = u_k(t) + \alpha L_t \epsilon_k(t+1)$$

Convergence Theorem:

With constant step size α

$$\limsup_{k\to\infty} \mathbb{E}[\|\tilde{\boldsymbol{u}}_k\|^2] \le \alpha \frac{d_2\left(\eta_1 + \eta_2(2\|\boldsymbol{y}_d\|^2 + \eta_3)\right)}{1 - \alpha\eta_4\lambda_{\max}(\boldsymbol{\Omega})} \frac{\lambda_{\max}(\boldsymbol{\Omega})}{\lambda_{\min}(\boldsymbol{\Omega})}$$

With decreasing step size α_k

$$\lim_{k\to\infty} \mathbb{E}[\|\tilde{\boldsymbol{u}}_k\|^2] = 0$$

General case

$\nu_k \not\equiv 0$

Testing signals

$$\theta_k^\circ(2i-1) = -\mu_k'(2i-1) + \nu_k'(2i-1)$$
$$\theta_k(t) = -1$$

$$\theta_k^\circ(2i) = \mu_k'(2i) + \nu_k'(2i)$$
$$\theta_k(t) = 1$$

Mean estimate of additive noise

$$\hat{\nu}_k = \frac{1}{n}\sum_{t=1}^{n} \theta_k^\circ(t)$$

Corrected testing signals

$$\theta_k^*(t) = (-1)^t[\theta_k^\circ(t) - \hat{\nu}_k]$$
$$= \mu_k'(t) + (-1)^t(\nu_k'(t) - \hat{\nu}_k)$$

MIE of multiplicative randomness

$$\widehat{\gamma_k} = \frac{\omega_h}{q_N}\prod_{t=1}^{N}\left(1 - \omega_h\theta_k^\circ(t)\right)$$

Convergence Theorem:

With constant step size α

$$\limsup_{k\to\infty} \mathbb{E}[\|\tilde{\boldsymbol{u}}_k\|^2] \le \alpha \frac{d_2\left(\eta_1^* + \eta_2^*\left(2\|\boldsymbol{y}_d\|^2 + \eta_3^*\right)\right)}{1 - \alpha\eta_4^*\lambda_{\max}(\boldsymbol{\Omega})} \frac{\lambda_{\max}(\boldsymbol{\Omega})}{\lambda_{\min}(\boldsymbol{\Omega})}$$

With decreasing step size α_k

$$\lim_{k\to\infty} \mathbb{E}[\|\tilde{\boldsymbol{u}}_k\|^2] = 0$$

总结

- **Known statistical information**: Three iterative averaging mechanisms for the input signal are established, which ensures the stability of the dynamic process of the system, and clearly depicts the trade-off relationship between the learning rate and the tracking accuracy (TAC 2021, TNNLS 2020, TSMC 2022).

- **Unknown statistical information**: A comprehensive gradient estimation method based on stochastic iterative differences (TNNLS 2021) and an iterative estimation mechanism based on test signals are established, which are used for the correction of biased information (TNNLS 2022).

- **Changing statistical information**: Characterize the minimum information requirement for achieving asymptotically accurate tracking, establish an unbiased estimation of the inverse of the mean based on only a single batch of limited test signals, and realize the extraction and correction of key information (TAC 2023).

- **Extensions**: Techniques and results can be extended to multiagent systems with fading communication (TNNLS 2023) and point-to-point tracking control via fading communication (TCYB 2024).

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Thanks

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